A MODEL ANALYSIS OF A LATERALLY LOADED PILE

by

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THESIS

Presented to the Faculty of the Graduate School of
The University of Texas in Partial Fulfillment
of the Requirements

For the Degree of
Master of Science in Architectural Engineering

THE UNIVERSITY OF TEXAS
AUGUST, 1957

PREFACE

Most engineers persist in attempts to create a method that will shorten and make more convenient the path to a solution of some problem. This study is such an attempt.

The writer wishes to thank Hudson Matlock for his aid, encouragement, and patience extended during the development of this study. The writer also acknowledges the aid and interest of Lymon C. Reese, and the interest extended by W. W. Dornberger.

Robert L. Thoms

Austin, Texas
August 10, 1957

TABLE OF CONTENTS

I

INTRODUCTION

The solution of a laterally loaded pile has received considerable study in recent years, and certain types of solutions are still undergoing research at the present time.

It is not the purpose of this chapter to review previous mathematical solutions of laterally loaded pile problems. However, a certain assumption employed by some mathematical solutions is pertinent to the concept of a model. In some mathematical solutions the soil supporting a pile is assumed to be similar to a series of elastic elements supporting the pile. The pile with this assumed elastic support is similar to an elastically supported beam.[1] This assumed elastic support of the pile may be illustrated, and often is, as a series of coil springs supporting the pile along its length. Perhaps this illustration of the assumed soil support should lead naturally to the concept of a model which would accomplish physically what other methods accomplish mathematically.

In the spring of 1956, the construction of a model which would make possible a solution of a laterally loaded pile was suggested as a thesis project. There began a

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[1] Biot, M. A. (1937). "Bending of an infinite Beam on an Elastic Foundation." Journal of Applied Mechanics, Vol. 4, pp. A1-A7.

series of model designs, on paper, indicating the possible arrangements of mechanical devices capable of representing a laterally loaded pile.

During the period of research on possible model designs, a paper was found that described models for predicting soil pressures on flexible foundations.[2] The described models for predicting soil pressures on flexible foundations employed a brass welding rod supported by rubber bands in opposing tension.

Some mechanical arrangements of the present model design were conceived on the basis of the described model arrangement used to predict the soil pressure on a flexible foundation. Because of the varying nature of the supporting soil reaction over the length of a laterally loaded pile, model supports with a variable spring constant were devised.

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[2]Bull, A. " Models for Determining the Pressure Distribution Along Flexible Foundations." Board of Transportation, City of New York.

II

PURPOSE AND METHOD

A The Problem to be Solved.

The solution of a laterally loaded pile with constant cross section and lateral support varying with location and pile deflection may generally be found by mathematical methods. The solution of a pile with varying cross section and lateral support has not been determined mathematically at this time to the knowledge of this writer. In all likelihood this problem too, will shortly fall before the ingenuity of engineers studying it. The purpose of this thesis is to provide another and perhaps more convenient solution of a laterally loaded pile with varying support, and to offer a tentative method of solution for a laterally loaded pile of varying cross section and support.

B Method of Solution.

As indicated earlier, the method by which the solution of a laterally loaded pile will be determined will be that of model analysis. The desired performance of the loaded model is that it shall have a deflection curve similar to the deflection curve of a laterally loaded pile supported by soil reactions. If the model is designed for this performance, there exists a constant relation between the deflection curve of the model and prototype which may be

determined by a comparison of flexural characteristics. By using this known constant relation and the measured deflection curve of the model, the deflection curve of the prototype may be predicted. The deflection curve of the prototype may then be transformed into soil pressures by employing known or assumed soil reaction curves. With the soil pressures on the pile predicted, a complete solution of a pile problem is possible.

III

DESIGN AND THEORY

A Proposed Model Design and Approximations.

To produce similar deflection curves under load, it is necessary for the prototype and model to possess certain properties determined by flexural relations. If these properties exist in the model in proper relation to the prototype and thereby produce similar deflection curves, the prototype and model are said to be flexurally similar and satisfy the necessary similitude requirements. The basis of the model design is therefore a case of representing flexural properties of the prototype by observing the requirements of flexural similitude. In addition, proper relations between model supports and the assumed properties of the soil must be maintained.

The prototype to be represented by the model consists of an elastic beam laterally loaded at one end by a concentrated force or moment or both; and supported along its length by a continuous soil reaction of a varying nature. Representation of the prototype then, can be resolved into a composite model; one part of which represents the elastic beam, and a second part which represents the supporting medium. Now the elastic beam or actual pile may be represented in the model easily and accurately by an elastic rod

with dimensions determined by similitude requirements. The soil reactions however, present a more complex problem since soils are not generally homogeneous over the length of a pile. Both the secant modulus of soil reaction and the ultimate resistance of the soil may vary at different depths of the pile.

In the following pages of this study frequent reference will be made to similar lengths and locations in the model and prototype. The ratio of the length of the prototype pile to the length of the model pile will be called the length scale. Any portion of the length of the model or distance to a location in the model, when multiplied by the length scale, establishes a similar portion of the length or distance to a location in the prototype.

Two approximations are made in representing soil conditions in the model. The first is with regard to the effect of a continuous support of a varying nature. The continuous nature of the soil's support may be approximated in the model by a series of concentrated supports, providing their number is sufficiently large in comparison to the length of the elastic member they support and shear between soil layers is negligible. If there are n supports in the model supporting an elastic rod of length L_m, each support will provide a reaction for a length

interval of $\frac{L_m}{n}$. Each concentrated support in the model may be thought of as the action of a soil pad of some length in the prototype. The soil pad in the prototype may be assigned to support a length of pile similar to the length of elastic rod supported in the model by a concentrated support. This determines the length of the soil pad represented by one concentrated model support as being $\frac{L_p}{n}$ in the prototype, where L_p is the length of the actual pile and n is the number of supports in the model. This approximation makes possible a practical representation of the continuous effect of the soil support, without an appreciable error.

The second approximation deals with the nature of the soil itself and its representation by the concentrated supports of the model. The necessary effect of the model supports may be illustrated in Figure 1 with similarly located sections of a pile and the model. It may be seen from Figure 1 that each model support represents a soil support having a soil reaction curve assumed constant over the length $\frac{L_p}{n}$. A satisfactory approximation of the soil reaction curve of each soil support must be furnished by the corresponding model support. The coordinates of the typical assumed soil reaction curves shown in Figure 1 are the soil resistance p and the lateral pile deflection, y. The soil resistance p is the resisting force, in

pounds, exerted by the supporting soil on a one-inch interval of pile length deflecting laterally.

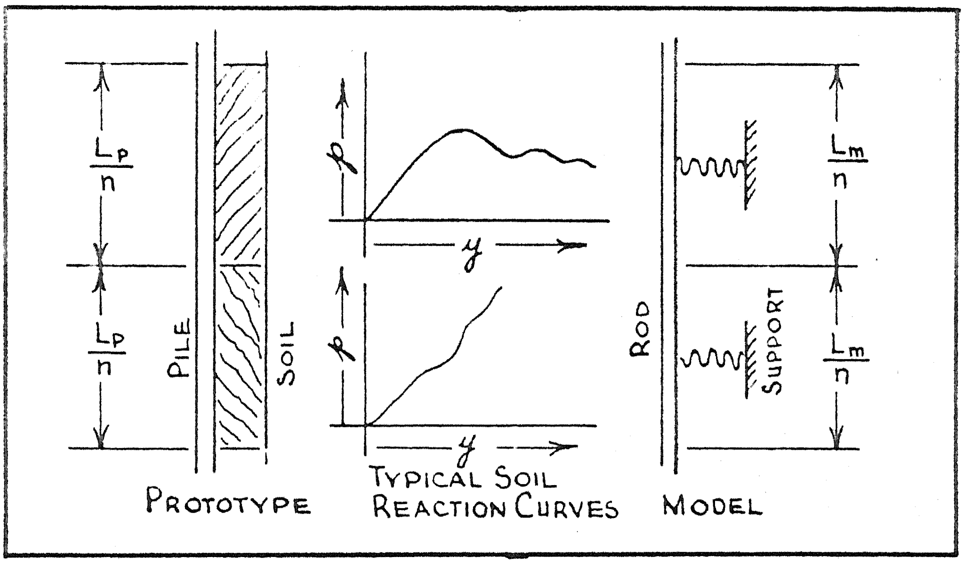


Figure 1

There are two methods by which the soil reaction curve of a soil support may be approximated. Method (1) is an approximation of the shape of the soil reaction curve by means of either; (a) an elastic support, or (b) a combination elastic and dead-weight support. The choice of an elastic or elastic and dead-weight support depends upon the shape of the assumed soil reaction curve. Method (2) is an approximation of the effective soil reaction curve as indicated by a secant modulus. These two methods may be examined separately and in more detail with the following explanations.

In explanation of method (1) of soil reaction

approximation, assume the solid line of Figure 2 to be a typical soil reaction curve constant over a soil support of length $\frac{L_p}{n}$ and applicable at some particular depth of a pile. The shape of the curve may be approximated by two straight lines as indicated with the dashed lines in Figure 2. If the typical soil reaction curve had been of a linear nature, a single straight line would have been adequate. By observing the proper similitude requirements, the dashed curve of Figure 2 may be reproduced in the model with an elastic support and dead weight. An all-elastic support in the model would represent a soil support with a linear soil reaction curve.

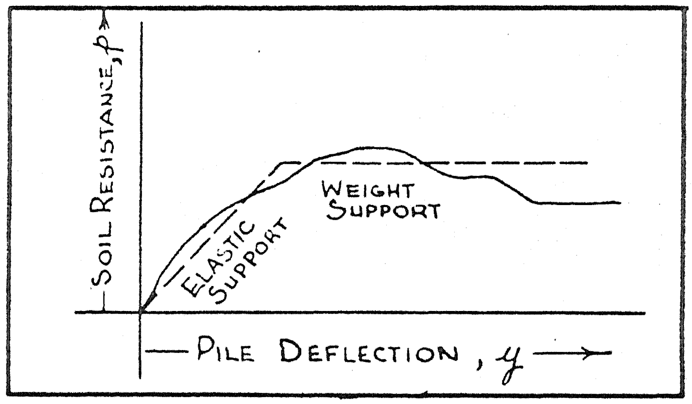


Figure 2

Method (2) of approximating a soil reaction curve involves the determination of an effective secant modulus. The typical soil reaction curve of Figure 2 is shown again as a solid line in Figure 3. Now, at any given deflection,

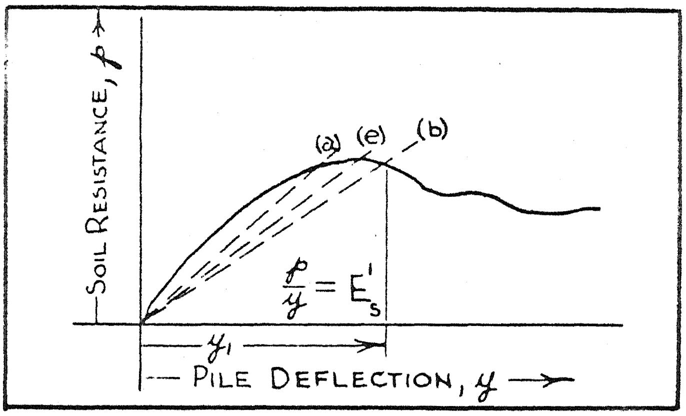


Figure 3

there exists some secant modulus of soil reaction, $E_s' = \frac{p}{y}$, that will give the effect of this soil reaction curve. The solution of the effective secant soil reaction modulus requires a trial and adjustment process of several supports of the elastic model. In order to determine E_s', a secant modulus may be assumed for the soil reaction curve in Figure 3 as indicated by the slope of the dashed line (a). On the basis of similitude requirements, the stiffness of the similar elastic support of the model may be adjusted so as to represent this assumed secant modulus. This operation would be carried out for all soil supports and their corresponding similar model supports. The model may then be loaded, and the deflection at the elastic supports found. The deflection of the model for the typical soil support under discussion may be transformed with similitude relations into an equivalent pile deflection, y_1.

The deflection, y_1 , when plotted on the soil reaction curve would give some new secant modulus as the slope of line (b) in Figure 3. The model elastic support may then be readjusted so as to reproduce the effect of this new secant modulus, and the previous procedure would be repeated. The process of trial and adjustment may be continued until the effective secant modulus, E_s' , is determined such that a readjustment of the model elastic support is not necessary. A readjustment is not necessary if the previously assumed secant modulus, in Figure 3 the slope of line (e), agrees satisfactorily with the secant modulus determined by the deflection of the model plotted on the soil reaction curve. The entire process as described for one typical soil support would be carried out simultaneously for all soil supports and their corresponding similar elastic model supports.

Method (2) may be done by mathematical methods, either by hand, or with a computer. However, with close spacing the computations are extremely tedious and furthermore, convergence is not always obtained. The function of the model is then to automatically and quickly make all beam calculations for each trial pattern of deflection.

In comparing the two methods of approximating the reaction curve of a soil support it may be observed that each

method has advantages and disadvantages. Method (1) makes possible a prediction of prototype deflection with the adjustment of model supports during one loading of the model. The physical application of changing from elastic model support to plastic model support as the deflections increase past the elastic limit is not as easily accomplished as maintaining an all-elastic model support. Method (2) requires only elastic support in the model, but it involves a process of trial loadings and support stiffness adjustments to determine the effective secant modulus of the soil reaction curve. Both methods will be referred to in the pages that follow.

With the model design proposed and with its approximations accepted, it becomes necessary to consider the concept of flexural similitude; and to define the constant relation between model deflection and prototype deflection called the prediction equation.

B The Prediction Equation.

In developing the prediction equations of this section and the flexural similitude requirements of the next section of this chapter, certain notations were used to refer to characteristics of the prototype and model. These notations are listed, and they will be employed in later pages.

Model	Prototype	
y_m	y_p	Deflection.
$E_m I_m$	$E_p I_p$	Flexural rigidity.
P_m	P_p	Applied lateral load.
M_m	M_p	Applied moment.
L_m	L_p	Length of pile.
x_m	x_p	Distance to a similar point.
	E_s	Soil reaction modulus as determined by either method (1) or (2) of section A, Chapter III.
j		Spring constant of an elastic support in the model.
n		Number of elastic supports in the model.

In Similitude in Engineering, Murphy finds the prediction factor of a distorted deflection model by comparing the computed deflections using the flexure theory.[3] The procedure in this problem is identical. A difficulty arises in that the theory for deflection of the beam under the indeterminate loading of this problem is not so easily established.

For any beam with an applied concentrated load and negligible shear deflection, the deflection due to bending

[3] Murphy, G. (1950). Similitude in Engineering, P. 122. New York: The Ronald Press Company

may be expressed as $y = \frac{PL^3}{\mu EI}$, where μ is a constant for similarly loaded beams at similar locations of deflection.[4] The term, similarly loaded beam, applies to the location of the applied load, and to the location and magnitude of the reactions.

The basis of design of the model is that the values of μ at corresponding points shall be constants for the model and prototype. If μ is constant for the prototype and model, then the ratio of model deflection to prototype deflection at similar locations will be:

$$(1) \quad \frac{y_m}{y_p} = \frac{\frac{P_m L_m^3}{\mu E_m I_m}}{\frac{P_p L_p^3}{\mu E_p I_p}}$$

Relation (1) simplifies to:

$$(2) \quad y_p = y_m \frac{\frac{P_p L_p^3}{E_p I_p}}{\frac{P_m L_m^3}{E_m I_m}}$$

The deflection of the model for some applied load may be measured at certain points. If the measured deflections of the model are substituted in relation (2), deflections of the prototype may be predicted at similar points.

The prediction equation for the model with an applied end moment may be developed in a fashion similar to that of

[4] Charlton, T. M. (1954). Model Analysis of Structures, p. 13. London: E. & F. N. Spon

the prediction equation for the model with an applied end load. The deflection equation of any elastic member with an applied end moment will be of the form, $y = \frac{ML^2}{\beta EI}$, where M is the applied end moment, and β is a constant similar to the previously discussed μ. If the model and prototype are flexurally similar such that μ is constant, then β is a constant also. The prediction equation for deflection at similar locations in prototype and model will be:

$$(3) \quad y_p = y_m \frac{\dfrac{M_p L_p^2}{E_p I_p}}{\dfrac{M_m L_m^2}{E_m I_m}}$$

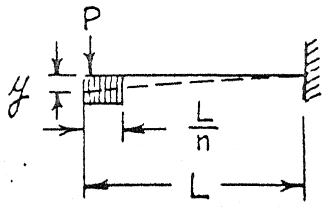
If the prototype is subjected to both an applied end load and moment, then the model may be loaded with both an applied load and moment which are in the same proportion as the load and moment on the prototype. Relations (2) and (3) may then be used in combination to predict the deflection of the prototype from the loaded model.

C Flexural Similitude Requirements.

In order for μ to be a constant at similar locations of deflection for both model and prototype, the applied load must be at similar locations, and the rod and pile must be supported by reactions similar in location and in behavior.

Now the length scale provided by the ratio of prototype length to model length has been observed for the location of (1) applied load, (2) spring supports representing various soil conditions, and (3) deflection measurements. It follows that the similitude requirements of length and location have been met. Next, the stiffness of the spring supports in the model may be determined by the following: If at any point a load is applied, and the ratio of the portion of the shear load carried by the prototype beam, to the applied load, is the same as in the model, then the prototype and model are similarly supported.

Example: A cantilever prototype with end elastic support.



$$y = \frac{P_B L^3}{3EI}$$

$$P_B = y\,\frac{3EI}{L^3}$$

$$P_B = P - P_S = E_S \cdot \frac{L}{n} \cdot y$$

WHERE P_B = PORTION OF SHEAR LOAD CARRIED BY BEAM.

P_S = LOAD CARRIED BY SOIL SUPPORT OF LENGTH, $\frac{L}{n}$.

If $\frac{P_B}{P_S}$ is maintained as a constant for all such similar beams it follows that $\frac{P_B}{P}$ will also be constant, and the beams will be similarly supported.

$$\text{NOW} \quad \frac{P_B}{P_S} = \frac{y_B \frac{3EI}{L^3}}{y_S\, E_S \cdot \frac{L}{n}} = \frac{\frac{3EI}{L^3}}{E_S \cdot \frac{L}{n}} \quad \text{SINCE} \quad y_B = y_S$$

It follows that for a cantilever model of known

material, cross section, length, and elastic support with stiffness j supporting a length similar to $\frac{L}{n}$, the following relation must be true for similitude of supports.

$$(4) \qquad \frac{\frac{3E_mI_m}{L_m^3}}{j} = \frac{\frac{3E_pI_p}{L_p^3}}{E_s\frac{L_p}{n}} \quad \text{OR} \quad \frac{\frac{E_mI_m}{L_m^3}}{j} = \frac{\frac{E_pI_p}{L_p^3}}{E_s\frac{L_p}{n}}$$

The cantilever in this example is a simplified case showing the relation which must exist between beam and elastic support. The final relation must be true for any elastically supported elastic beam in order to obtain similitude of reactions.

Assume Method (1-b) of approximating soil reaction as discussed earlier in this chapter is followed. When the action of a soil support on the prototype beam is no longer elastic and plastic behavior occurs, similar effects may be reproduced in the model and deflection analyzed on the basis of continued flexural similitude between model and prototype. The plastic action occurs at certain depths on the prototype pile when limiting soil stresses are reached. The limiting stress may be transformed into a limiting load for a soil support over some length of pile, $\frac{L_p}{n}$. This limiting load could be expressed as a product of a coefficient, C, and the applied load on the pile. Now the same effect may be reproduced in the

model by hanging some dead weight at a location similar to that of the soil support in plastic action on the prototype. The weight will have a magnitude equal to the product of the coefficient, C, and the load applied to the model. The value in determining the plastic load in terms of a coefficient and the applied load is realized when a distorted load is applied to the model for more easily read deflections. Thus, similar loading conditions are preserved in the model and relation (2) will provide the prediction factor for deflection of the prototype. The limiting load of elastic action at a spring support in the model may be transformed into a more easily observed limiting deflection by means of the spring constant, j .

Assume method (2) of approximating the soil reaction curve, as determined by an effective secant modulus, is followed. Relation (5), which is another form of relation (4), may be used to reproduce the soil support moduli in the spring constants of the model supports. This relation will hold true throughout the trial and adjustment process required to determine all effective secant moduli.

$$(5) \quad j = \frac{E_m I_m}{L_m^3} \cdot E_s \cdot \frac{L_p}{n} \cdot \frac{L_p^3}{E_p I_p}$$

If the model and prototype have established dimensions, then the relation (5) may be expressed as relation (6).

$$(6) \quad j = J \cdot E_s \quad \text{WHERE} \quad J = \frac{E_m I_m}{L_m^3} \cdot \frac{L_p^3}{E_p I_p} \cdot \frac{L_p}{n}$$

It should be noted that relations (5) and (6) also apply to spring adjustments for method (1-a) and for the elastic range of the approximate soil reaction curve of method (1-b).

Up to now the discussion of model and prototype relations has been confined to the constant moment of inertia case of the flexural member. However, there is no reason to limit the model to this condition. Flexural similitude will continue to exist between a prototype of varying moment of inertia and a model if the proper similitude requirements are met. If the previously discussed requirements are met, the one new requirement is that there must be a factor which is constant between the moments of inertia of the model and prototype at similar locations of length.[5] The similitude requirements may be met by the following procedure. If the prototype and model are each assumed to have a constant moment of inertia equal to their least actual moment of inertia, the required spring constants may be found on the basis of relation (5). Now the moment of inertia of the model pile may be increased at locations

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[5]Murphy, G. op. cit., p. 119

similar to those locations in the prototype where increases of moment of inertia exist. The relation, $I_m \nu = I_p$, must be true at all similar locations. The constant ν may be determined by dividing the least moment of inertia of the prototype by the least moment of inertia of the model.
The prediction factor may be found by a modification of relation (2). With flexural similitude preserved the relation will still hold true provided that the moment of inertia of the model and prototype used in the relation are at similar locations. This may be justified by applying area-moment principles and considering deflection as a function of the moment of the $\frac{M}{EI}$ diagram. If the moment diagrams of prototype and model are similar it follows that the $\frac{M}{EI}$ diagram will be similar when $\frac{I_p}{I_m}$ is constant at all similar locations. It should be noted that moments of inertia only are used in maintaining similitude requirements between cross sections of the prototype and model. Therefore, the model is not limited to a cross section geometrically similar to the cross section of the prototype.

IV

DESCRIPTION OF A MODEL AND ITS OPERATION

A. Description of The Model.

The model which was developed and constructed for this study consists of an elastic rod representing a pile and a series of adjustable coil spring supports representing the soil support. To simulate plastic soil reactions, dead weights may be used to replace the coil springs.

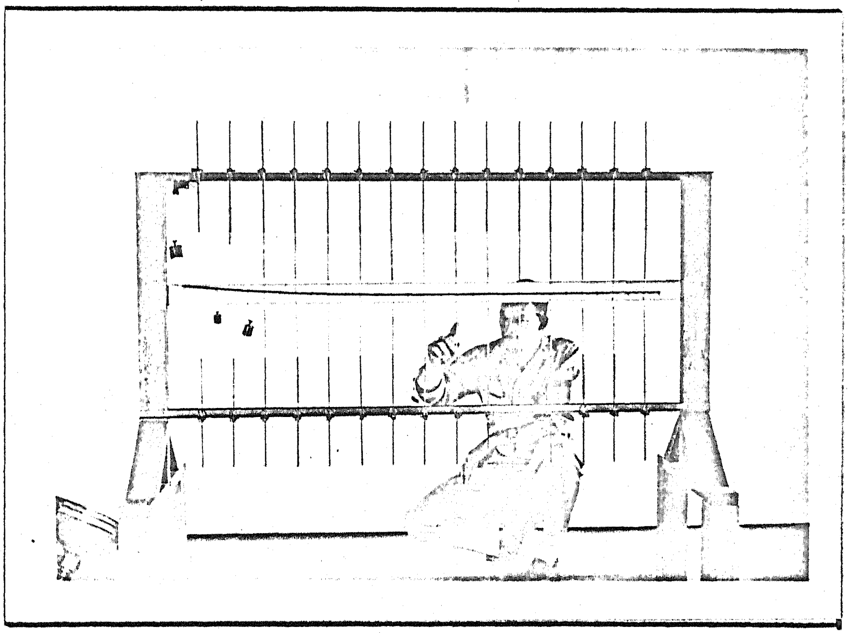


Figure 4

Figure 4 shows the model with a load applied at one end. The elastic rod seen in the center of the supporting frame of the model represents the pile. Fifteen pairs of opposing coil springs at four-inch intervals support the

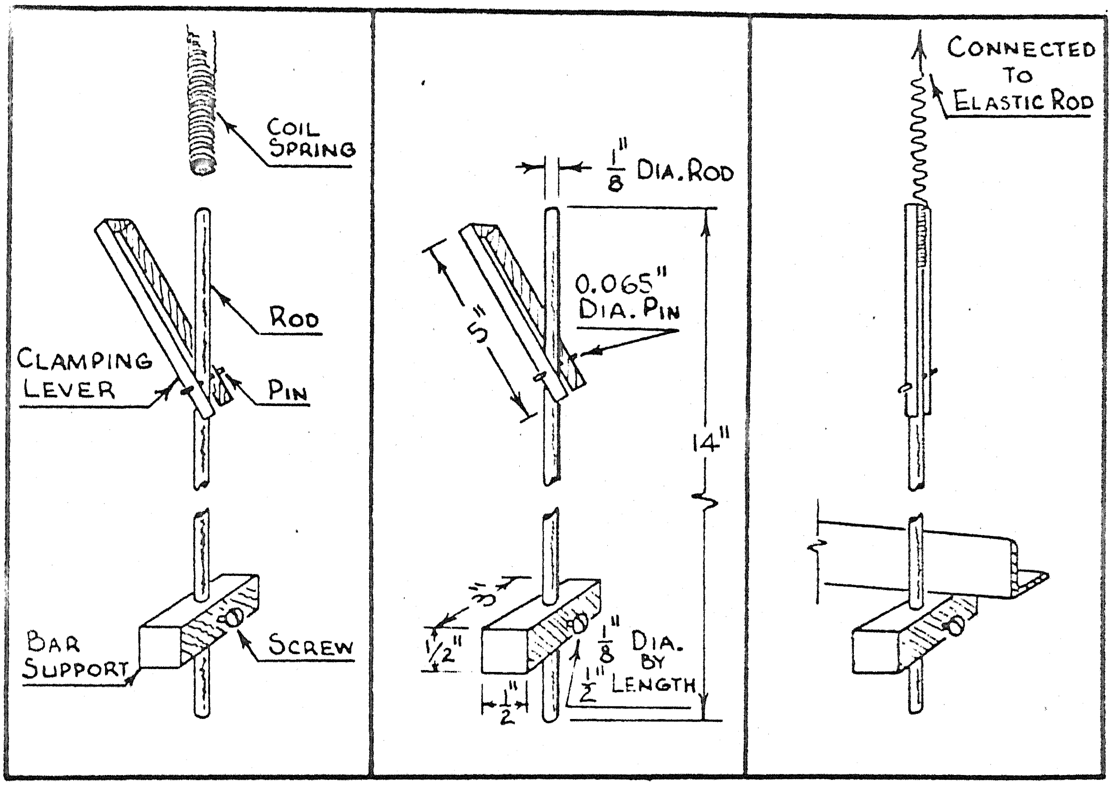
elastic rod. Each coil spring is connected to the supporting frame of the model by a device capable of changing the length of the coil spring supporting the elastic rod. The supporting frame of the model is composed of relatively rigid wooden members and steel angles.

The elastic rod is of steel and is sixty inches in length. The present rod is 3/16 inch in diameter.

The steel coil springs supporting the elastic rod are 4.75 inches long and 0.22 inch in outside diameter. Each spring contains two hundred coils and has a measured spring constant of 0.606 pounds per inch of deflection with the full length of the coil spring free to extend. The spring constant may be varied by changing the length of the coil spring supporting the elastic rod.

The device used to connect a coil spring to the supporting frame of the model is shown in Figure 5(a). The dimensions of the device are shown in Figure 5(b). The operation of this device in changing the length of a coil spring supporting the elastic rod will be described in the next section with the aid of Figure 5(c). Figure 6(b) shows rubber bands maintaining the positions of the connecting devices on the supporting frame of the model.

The supporting frame of the model is made up of four lengths of 1 by 1 by 1/8-inch steel angle and two lengths



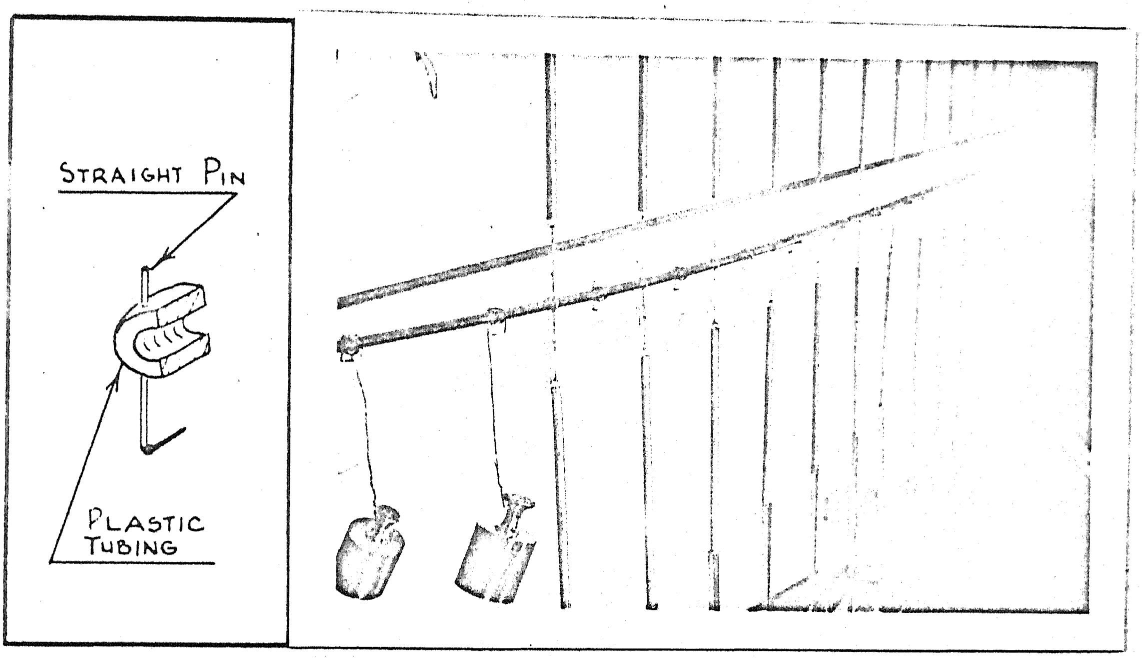
(a) (b) (c)

Figure 5

of nominal 2 by 4 wooden members. The angles are placed back to back in pairs and separated by the wooden members as seen in Figure 6(b). Outside dimensions of the supporting frame of the model are 31 inches by 6 feet. Connections are made at the corners by two 1/4-inch diameter bolts, 2-1/4 inches long.

Deflection measurements may be taken with the aid of the pointers midway between coil spring supports. Each pointer device, shown in Figure 6(a), is made of a short length of plastic tubing and a bent straight pin. The

short lengths of plastic tubing are split and the straight pins imbedded in the wall of the tubing opposite the split. The pointer devices clamp on the elastic rod by virtue of the elastic nature of the plastic tubing.



(a) (b)

Figure 6

The grid immediately behind the elastic rod, seen in Figure 6(b), consists of grid paper fastened to a 1 by 2-3/4-inch wooden member parallel to the unloaded elastic rod. The grid paper is ruled with twenty lines per inch.

B Description of Operation of The Model.

To cover a range of pile problems, there are four things which can be changed, (1) the diameter of the elastic rod to represent the pile, (2) the length of the elastic rod

to represent the pile, (3) the stiffness of the coil spring supports, and (4) the number of coil spring supports. The following description deals only with the physical operation of the present model and assumes that the preceeding variables have been established. Section B of Chapter V gives a description of rod selection and spring constant determination on the basis of the flexural similitude requirements of Chapter III.

The connecting device shown in Figure 5 may be used to obtain the established spring constants by changing the lengths of the coil springs supporting the elastic rod. The rod of the device is of such a diameter that it will fit inside a coil spring. When the clamping lever is closed and parallel to the rod, as shown in Figure 5(c), it passes between spring coils at the end of the rod and transmits the load from the loaded portion of the spring to the rod. The length of spring supporting the elastic rod may then be changed by opening the clamp, sliding the coil spring such that the desired length is above the connection, and then closing the clamping lever.

To keep proper tension in the coil springs, the rods of the connecting devices may be slid through the supporting bars that bridge the two angles of the supporting frame of the model. The rod, of a connecting device, may be fixed at a position that maintains proper tension in the coil

spring by a tightening of the screw in the support bar against the rod.

The model may then be loaded by a weight over a pulley as seen in Figure 4, or by a free hanging weight if it is not necessary to replace the elastic model supports with dead weights. The applied weight may be of any magnitude that does not cause a closure of any coil spring support.

Deflections of the loaded elastic rod may be found as vertical differences in the unloaded and loaded positions of the pointers clamped on the elastic rod. The vertical positions of the pointers on the grid behind the elastic rod may be noted with an engineer's transit approximately fifteen feet from the model and at the height of the model. The function of the transit is to eliminate reading parallax.

The operation of the model may then be summarized in the following series of steps;

(1) adjustment, with the connecting device, of the coil spring supports to give established spring constants,

(2) adjustment of tension in the coil springs with the supporting bars of the connecting devices,

(3) preload reading of pointer positions with a transit,

(4) a loading of the model,and

(5) a reading, with the transit, of pointer positions of the loaded model.

V

EXAMPLE OF USE OF THE MODEL

A An Example Pile Problem and a Check on the Success of The Model.

To explain the practical application of the model and to show to what degree of accuracy the model operates, a fictitious pile problem was attacked. The data of the fictitious pile problem was obtained by choosing values for the prototype that would require the action of the full lengths of the coil springs in the model supports and the dimensions of the elastic rod in the present model developed for this study. An actual pile problem could have been solved by a modification of the use of the model described in this section. Section B of this chapter describes the use of the model in solving pile problems that vary from the fictitious example of this section.

Prototype data:

$I_p = 1500 \text{ in.}^4$

$E_p = 30 \times 10^6 \quad \therefore E_p I_p = 4.5 \times 10^{10} \text{ in.-lbs}^2$

$L_p = 50 \text{ ft.} = 600 \text{ in.}$

$E_s = 368 \text{ lbs/in. per inch}$

$P_p = 30 \text{ kips}$

In this example the length of the elastic rod and the number of coil spring supports were assumed fixed for the present model. The available and assumed

characteristics of the model are listed.

Model data:

$$E_m = 30 \times 10^6 \text{ lbs/in.}^2$$
$$L_m = 5 \text{ ft.} = 60 \text{ in.}, \therefore \text{ Length Scale} = \frac{600}{60} = 10$$
$$n = 15 \text{ Coil Spring Supports}$$
$$j = 0.606 \text{ lb/in. Minimum}$$

Since the length of the rod and the number of coil spring supports were fixed in the present model, there remained two variables that allowed adjustment of the model until similitude existed between the model and prototype. These two variables were the spring constant, j , and the moment of inertia of the elastic rod, I_m .

In the example problem the moment of inertia of the elastic rod to represent the pile was selected on the basis of the similitude requirements as determined by relation (4) of Chapter III. The minimum spring constant of the coil spring supports was substituted in relation (4) in order that the entire lengths of the coil springs would be utilized in supporting the elastic rod.

$$(4) \quad \frac{\frac{E_m I_m}{L_m^3}}{j} = \frac{\frac{E_p I_p}{L_p^3}}{E_s \cdot \frac{L_p}{n}} \quad \text{or} \quad I_m = \frac{E_p I_p \cdot j \cdot L_m^3}{L_p^3 \cdot E_m \cdot E_s \cdot \frac{L_p}{n}}$$

$$I_{m_{REQ}} = \frac{(4.5 \times 10^{10})(0.606)(60)^3}{(600)^3(30 \times 10^6)(368)(40)} = 6.1 \times 10^{-5}$$

Since the desired cross section of the rod to be selected was circular:

$$\frac{\pi D^4}{64} = 6.1 \times 10^{-5} \quad \therefore \quad \text{DIA.}_{\text{REQ}} = 0.187'' \text{ OR } \frac{3}{16}''$$

A 3/16-inch-diameter rod was then used to represent the pile on the basis of the preceeding calculation. The constant soil reaction modulus of 368 lbs/inch per inch of pile was represented in the coil spring supports by the minimum spring constant of 0.606 lbs/inch. Therefore the coil spring supports were adjusted with the connecting device described earlier such that the entire lengths of the coil springs would support the elastic rod.

The load applied to the model, equivalent to the applied load on the pile, may be of any magnitude that will produce easily read deflections without a closure of the coil spring supports. The distortion of an applied load on the model is taken into account in the prediction equation.

With the model adjusted to represent the assumed pile and soil reaction, a lateral concentrated load of 1.41 lb was applied to one end of the elastic rod. The load of 1.41 lb was found to produce easily read deflections at the pointers and did not cause a closure of any coil springs in the model supports.

The deflection of the loaded elastic rod was found

along the length of the rod as vertical differences of preloaded and loaded pointer positions. The positions of the pointers were estimated on the grid immediately behind the elastic rod to the nearest one hundreth of an inch with the aid of a transit equipped with a closeup lens. The transit was placed at the same height and approximately fifteen feet from the model on a line perpendicular to and bisecting the length of the model. The vertical positions of the pointers along the entire length of the rod were noted with this arrangement, and the position of the transit was not changed.

Figure 7 shows the locations of the pointers from the loaded end of the rod, and the values of deflection used in the steps taken to transform pointer deflections into equivalent forces exerted on the elastic rod.

The deflections found for the loaded elastic rod at the positions of the pointers are shown tabulated in the second column of Figure 7.

In the present model a pointer exists two inches from and on either side of each coil spring support. Therefore, a significant error is not introduced by assigning to a support a deflection obtained as an average of the two indicator deflections on either side of the support.

The third column of Figure 7 is the tabulation of deflections obtained for the coil spring supports by averaging the

(1) Distances from loaded end of rod	(2) Pointer deflections	(3) Support deflections	(4) Forces exerted by supports
inches	inches	inches	pounds
0	-1.25		
2		-1.09	0.66
4	-0.92		
6		-0.77	0.47
8	-0.62		
10		-0.51	0.31
12	-0.41		
14		-0.30	0.18
16	-0.19		
18		-0.13	0.08
20	-0.07		
22		-0.03	0.02
24	0.01		
26		0.04	-0.02
28	0.06		
30		0.07	-0.04
32	0.08		
34		0.08	-0.05
36	0.09		
38		0.09	-0.06
40	0.08		
42		0.07	-0.04
44	0.06		
46		0.05	-0.03
48	0.05		
50		0.04	-0.02
52	0.03		
54		0.02	-0.01
56	0.00		
58		0.00	0.00
60	-0.01		

Figure 7

deflections of the two neighboring pointers.

The deflections of the coil spring supports were then transformed into forces acting on the elastic rod by multiplying the support deflections by the spring constant of

0.606 lb/in. Column (4) of Figure 7 presents the forces acting on the elastic rod as determined by the deflections of column (3) and the known spring constant.

In ordinary use of the model the deflection of the coil spring supports could have been transformed immediately into deflection of the pile rather than into forces acting on the elastic rod. However, the forces acting on the elastic rod were necessary for the check on the model's performance that followed later. The deflection of the prototype at locations similar to the locations of the coil spring supports on the model are shown in Figure 8. The prototype deflections were obtained with prediction equation (2) of Chapter III. An example of prototype deflection prediction follows. Relation (2) is repeated for convenient reference.

$$(2) \qquad y_p = y_m \frac{\dfrac{P_p L_p^3}{E_p I_p}}{\dfrac{P_m L_m^3}{E_m I_m}}$$

At $x_m = 10$ in. in the model, and at $x_p = 10$ in. $\times 10 = 100$ in. in the prototype;

$$y_p = 0.51 \frac{\dfrac{(30)(600)^3}{4.5 \times 10^{10}}}{\dfrac{(1.41)(60)^3}{1850}} = 0.51(0.876) = 0.45 \text{ in.}$$

The deflections of the prototype thus obtained were transformed into pressures exerted by soil reaction on the

(1) Distance from loaded end of pile	(2) Soil support deflections	(3) Soil support reactions
inches	inches	kips
20	-0.95	14.0
60	-0.67	9.9
100	-0.45	6.6
140	-0.26	3.8
180	-0.11	1.6
220	-0.03	0.4
260	0.04	-0.6
300	0.06	-0.9
340	0.07	-1.0
380	0.08	-1.2
420	0.06	-0.9
460	0.04	-0.6
500	0.04	-0.6
540	0.02	-0.3
580	0.00	0.0

Figure 8

prototype by employing the known soil reaction moduli. The soil pressures shown in Figure 8 were obtained as a product of the soil reaction modulus, the deflection for a soil support, and the length of a soil support.

Example: At one hundred inches from the loaded end of the pile;

$$(E_s)(y)\left(\frac{L_p}{n}\right) = 368 \times 0.45 \times \frac{600}{15} = 6{,}620^{\text{lb.}}$$

A complete solution of the pile could have been then obtained by statics from the predicted soil pressures

acting on the pile at established locations as shown in Figure 8. Errors present in the prediction, by the model, of the soil pressures acting on the pile would have been smoothed in the process of integration required to obtain the moment diagram for the pile. Therefore, the deflection of the prototype was predicted with sufficient accuracy by the model. Instead of a solution of the pile, a check was next obtained on the performance of the model.

The elastic rod was considered as a beam loaded with the laterally applied end load of 1.41 pounds and the loads of the deflected coil spring supports as shown in column (4) of Figure 7. The shear diagram for the loaded rod is shown in Figure 9. The measured loading on the elastic rod was made statically correct by minor changes in the shear diagram. The revised values of shear that were necessary for static loading conditions are shown as the values in parentheses in Figure 9. The minor changes made in the shear diagram may be justified by considering the smoothing effect of the integration process that followed.

By the cumulative summation from the loaded end of the rod of the area under the revised shear diagram, the moment diagram of Figure 9 was obtained.

The deflection of the loaded elastic rod at the locations of the coil spring supports was calculated with the moment-area method on the basis of the moment diagram of

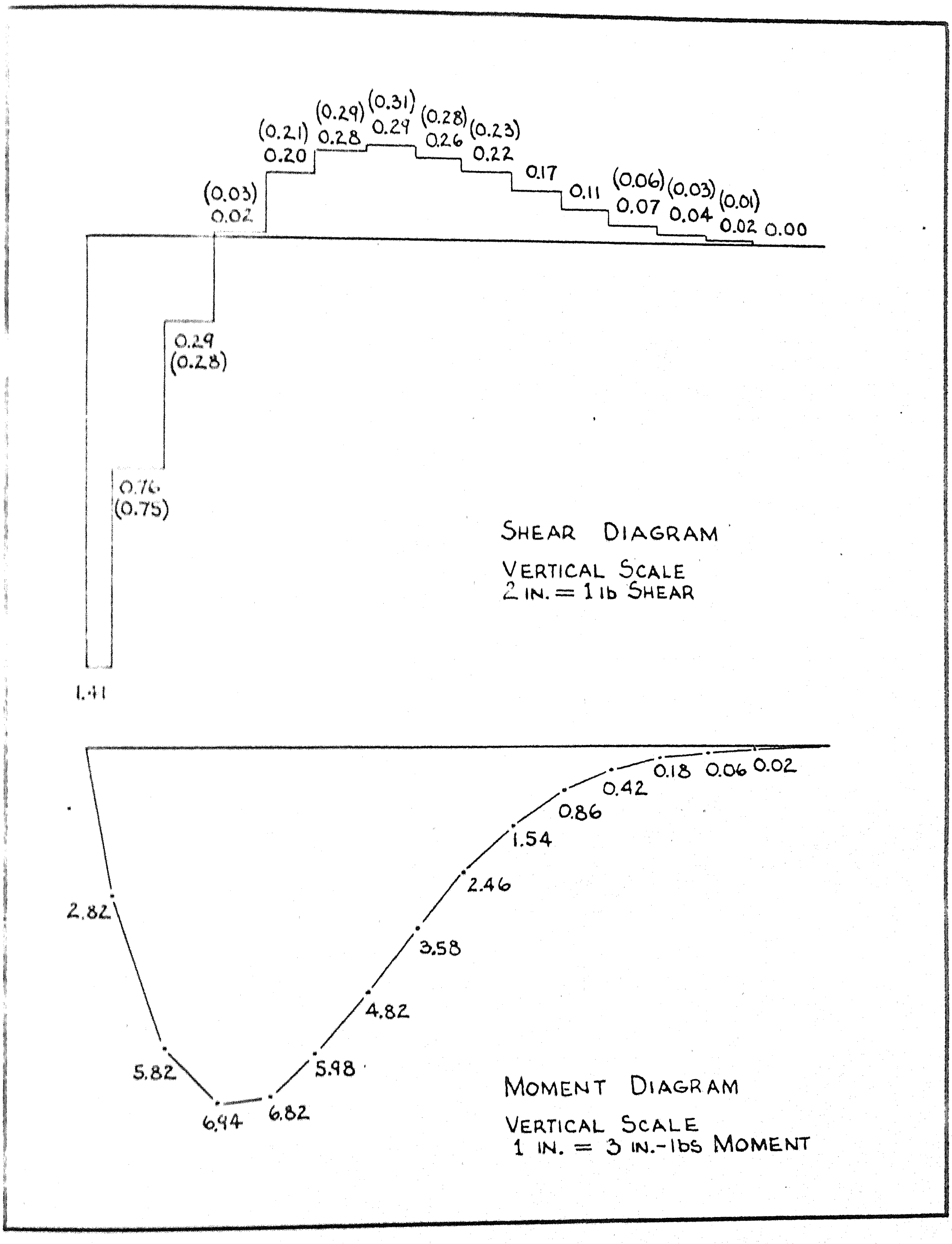

(0.03)
0.02
(0.21)
0.20
(0.29)
0.28
(0.31)
0.29
(0.28)
0.26
(0.23)
0.22
0.17
0.11
(0.06)
0.07
(0.03)
0.04
(0.01)
0.02
0.00
0.29
(0.28)
0.76
(0.75)
1.41
SHEAR DIAGRAM
VERTICAL SCALE
2 IN. = 1 lb SHEAR
2.82
5.82
6.94
6.82
5.98
4.82
3.58
2.46
1.54
0.86
0.42
0.18
0.06
0.02
MOMENT DIAGRAM
VERTICAL SCALE
1 IN. = 3 IN.-lbs MOMENT

Figure 9

Figure 9. Moments were taken about each support of the $\frac{M}{E_m I_m}$ diagram from the last support on the unloaded end of the rod. By comparing this calculated elastic curve to the measured elastic curve, the tangent to the measured elastic curve was found at the position of the last support on the unloaded end of the rod. Figure 10 shows the differences in measured and calculated deflections at the same locations on the elastic rod. By determining the tangent to the elastic curve, at the last support, as the slope of the straight line through the plotted points; minor errors indicated as deviations of some points from the straight line were neglected.

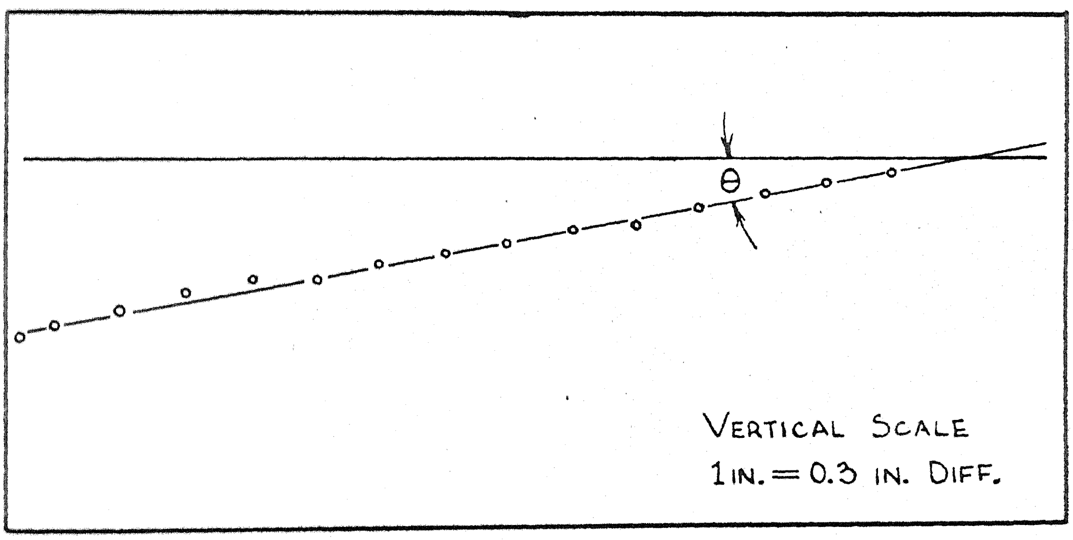


Figure 10

The tangent to the calculated deflection curve, at the last support on the unloaded end of the rod, was then made to agree with the determined tangent, at the same location,

to the measured deflection curve by rotating the calculated deflection curve about the last support on the unloaded end of the rod in a clockwise direction through the angle θ of Figure 10.

The ordinates of the properly oriented calculated deflection curve at the coil spring supports are tabulated in Figure 11 along with the ordinates, at the same locations, of the measured deflection curve. Column 4 of Figure 11 shows the differences in the ordinates of the measured and calculated deflection curves. It may be noted that the scatter in differences from point to point is small.

(1) Location from loaded end of rod	(2) Measured deflection	(3) Calculated deflection	(4) Differences in deflections
inches	inches	inches	inches
0	-1.25	-1.26	0.01
2	-1.09	-1.09	0.00
6	-0.77	-0.76	-0.01
10	-0.51	-0.49	-0.02
14	-0.30	-0.28	-0.02
18	-0.13	-0.13	0.00
22	-0.03	-0.02	-0.01
26	0.04	0.04	0.00
30	0.07	0.07	0.00
34	0.08	0.08	0.00
38	0.09	0.08	-0.01
42	0.07	0.07	0.00
46	0.05	0.05	0.00
50	0.04	0.04	0.00
54	0.02	0.02	0.00
58	0.00	0.00	0.00
60	-0.01	0.00	-0.01

Figure 11

B Variations of The Example Problem.

Had the example pile problem of this Chapter been more complicated, some variations in the described adjustment procedure of the model to represent the pile would have been necessary. Two complications of the example pile problem are possible in pile problems actually encountered. These two complications are, (1) the soil supports may vary in stiffness and linearality over the length of a pile and generally do, and (2) the cross section of a pile may vary in dimension at points along the length of the pile. The adjustment of the model to represent these two possible variations from the example pile problem will be discussed in the following paragraphs.

The selection of an elastic rod to represent a pile subjected to a soil reaction which varies over the length of the pile may be carried out with a modification of the previously described rod selection procedure of the example pile problem. By substituting the least soil reaction modulus encountered in the prototype and the minimum spring constant of the coil spring supports into relation (4), the moment of inertia of a theoretical elastic rod that will offer the most flexibility for later spring adjustments may be obtained. The moment of inertia thus determined may be increased until the dimensions of the rod become practical. The selection of the smallest cross section

of a rod to represent a pile with varying cross-section may be similarly treated. I_m and I_p of relation (4) will then apply to the moments of inertia for the smallest cross sections of prototype and model. The remaining cross sections of the rod to be sized may be found by the requirement that $\frac{I_m}{I_p}$ is a constant at similar locations in the prototype and model.

The spring constants required of the model supports to represent the varying soil reaction moduli over the length of a pile may be found with relation (6) of Chapter III. Assuming that the cross section of the elastic rod to represent the pile has been selected, relation (6) becomes a more convenient form of relation (4) for determining spring constants required of model supports in representing similarly located soil supports.

$$(6) \qquad j = J \cdot E_s \quad \text{WHERE} \quad J = \frac{E_m I_m}{L_m^3} \cdot \frac{L_p}{E_p I_p} \cdot \frac{L_p}{n}$$

The soil reaction modulus, E_s of relation (6) may be one of two approximations of soil reaction described in section A of Chapter III. Method (1) of approximating soil reaction described in Chapter III will employ relation (6) for the elastic portion of the approximate soil reaction curve. If the plastic portion of the approximate soil reaction curve is reached as designated by a limiting

deflection of a coil spring support, then a dead-weight will replace the coil spring support at the former location of the spring support on the elastic rod. The magnitude of the dead-weight may be determined by section C of Chapter III dealing with flexural similitude requirements. As method (2) of approximating soil reaction described in Chapter III requires only elastic action of the model supports, relation (6) will apply at all times to the model supports.

With the elastic rod selected to represent the actual pile and the coil spring supports adjusted to represent the soil reaction varying in character over the length of the pile, the remaining operational procedure of the model becomes identical to the described procedure of the example problem.

VI

CONCLUSION

The model developed for this study is capable of solving a variety and range of laterally loaded pile problems. Among the characteristics of pile problems that may be reproduced in the model are; (1) a pile loaded with an applied load or moment, or both, (2) a pile subjected to an approximated soil reaction varying with the depth of the pile and the applied load, and (3) a pile with varying cross section. The model is capable of solving a pile problem that contains any combination of the above characteristics.

The value of the model is realized when the effort spent on a pile problem solution obtained by mathematical methods is compared to the effort spent on the same solution obtained by the use of the model. Pile problem solutions that, by mathematical methods, require either a large amount of tedious labor by hand or the use of a computer, may be quickly and easily found with the model developed for this study.

VITA

Robert Lee Thoms was born in Nixon, Texas, on May 10, 1933, the son of Aletha Thoms and William F. Thoms. After completing his work at Nixon High School in 1950, he entered Texas Lutheran College in Seguin, Texas. He transferred to The University of Texas in June of 1952, and received the degree of Bachelor of Science in Architectural Engineering from The University of Texas in January of 1955. For the following two semesters he taught in the Civil Engineering Department at The University of Texas. He resigned this position in January of 1956 in order to work toward a higher degree. During the school year of 1956-57, he taught mathematics at Del Mar Junior College in Corpus Christi, Texas, then returned to The University of Texas in the summer of 1957.

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